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(6) SOLUTION OF STOCHASTIC CAPITAL
BUDGETING PROBLEMS IN A MULTIDIVISIONAL FIRM

BY

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1. Introduction

This paper considers a capital budgeting problem in a multi-divisional firm consisting of K divisions and headquarters (HQ). Each of the divisions has a set of physical projects which it can consider for adoption whereas HQ has access to the external financial markets.

It is assumed that HQ is responsible for developing an optimal divisional allocation of capital assets but that its information about the divisions' opportunities is incomplete. Several authors (see, for example, Carleton et al (3), Maier and Vander Weide (20), and Rosenblatt and Freeland (26)) have developed procedures which allow HQ the determination of a solution for its planning problem by an iterative exchange of information with the divisions. This exchange is organized according to rules that have been derived from mathematical decomposition (see Geoffrion (11) or Lasdon (18) for overviews).

However, the procedures are based on the assumption that the divisions know the future net cash flows that will be associated with the realization of divisional projects with certainty and that HQ's information about each of the division's opportunities is limited to a few series of net cash flows that correspond to feasible divisional programs.

The main purpose of this paper is to show how the organization of the communication process between HQ and divisions can be adapted to situations where the divisions' information about future net cash flows that are associated with single projects is stochastic.

This procedure is further generalized to allow HQ to formulate stochastic "estimates" (i.e. to quantify subjective probability distributions) for these net cash flows or for net cash flows that can be associated with aggregates of divisional projects. For the consideration of these stochastic estimates we will basically follow an approach developed by Freeland and Schiefer (10) for the solution of deterministic resource allocation problems. However, while the incorporation of HQ's stochastic estimates is primarily aimed at improving the efficiency of the communication process we will show that it might enable HQ to deal with situations where it suspects the divisions of doctoring the information submitted about their opportunities.

It is assumed that the communication process is basically organized according to rules that have been developed in the decomposition principle of Dantzig and Wolfe (7). This scheme implies a decision-making system where HQ is able to force the divisions to realize divisional programs that correspond to its solution for the firm's capital budgeting problem. However, procedures that refer to decision-making systems where the divisions are supposed to decide about their final programs themselves (see, for example, Tenkate (31) or Maier and Vander Weide (20)) could be used as a basis for the organization of the iterative information exchange as well.[1]

The ensuing formalized approach will comprise stochastic mathematical programming models. However, we will discuss its realization in a linear programming framework. In this context it should be noted that, while the communication process is presented with regard to decentralized decision making, its basic rules might be interpreted in mathematical decomposition as a method for solving (on a linear programming basis) capital budgeting problems that are large and stochastic.

2. A Capital Budgeting Problem in the Multidivisional Firm

Suppose the planning activities of HQ in the multidivisional firm are aimed at the maximization of the firm's horizon value subject to constraints on the realization of financial and physical divisional projects.[2]

A programming formulation for such a problem has been presented by Maier and Vander Weide (20). We use their approach and assume that if HQ had complete information about the firm's capital budgeting problem, its planning problem could be expressed mathematically as:

$$\text{maximize } z_{H1} = \sum_{k=1}^{K} a_k x_k + v_T - w_T$$

$$- \sum_{k=1}^{K} a_{1k} x_k + v_1 - w_1 \leq D_1$$

$$- \sum_{k=1}^{K} a_{tk} x_k - (1+r_{t-1}) v_{t-1} + v_t + (1+r'_{t-1}) w_{t-1} - w_t \leq D_t \quad (t=2,..,T) \tag{1}$$

$$0 \leq x_k \leq 1 \qquad (k=1,..,K)$$

$$v_t, w_t \geq 0 \qquad (t=1,..,T)$$

where

x_k = vector of decision variables, each representing the fraction of a project whose realization is part of division k's decision authority;

a_k = vector representing the horizon values of post horizon cash flows that are associated with one unit of x_k;

a_{tk} = vector representing the net cash flows in period t associated with one unit of x_k;

v_t = single period lending opportunity in period t;

w_t = single period borrowing opportunity in period t;

r_t = lending rates in period t;

r'_t = borrowing rates in period t;

D_t = total amount of net cash flows in period t that are associated with projects initiated prior to the start of the planning process.

However, it is assumed that neither HQ nor the divisions know the future "true" values of all of the net cash flows and lending/borrowing rates in the problem formulation (1) with certainty but that they possess information which might allow the formulation of random variables as stochastic estimates.

Basically, HQ is assumed to have information about its own decision variables v_t and w_t (t=1,..,T) as well as about projects that have already been realized, i.e., HQ is assumed to be able either to determine the true values or to formulate (subjective) probability distributions for estimates of r_t, r'_t and D_t, i.e., to formulate random variables $\bar{r}_t$, $\bar{r}'_t$ and $\bar{D}_t$. In addition, however, HQ's information about the firm's capital budgeting problem might allow the formulation of random variables $\bar{a}_{tk}$ and $\bar{a}_k$ (t=1,..,T;k=1,..,K) as stochastic estimates for future net cash flows and horizon values of post horizon cash flows that are associated with divisional projects or aggregates of divisional projects as represented by vectors $\bar{x}_k$ (k=1,..,K).

On the basis of this information HQ might formulate a first approach to the firm's capital budgeting problem. However, as at least parts of HQ's information is stochastic, a complete representation of its initial decision problem has to reflect its attitude towards the uncertainty in the problem .

To consider HQ's attitude in programming models a variety of different "decision rules" have been developed. For surveys see, for example, Kall (16) or Vajda (33). These rules cover mainly situations where

a) the probability distribution of the objective variable is of interest (see, for example, Markowitz (21), or the stochastic programming approach developed in Sengupta (30) or Tintner (32)), or where

b) the fulfillment of the budgetary constraints is required with certainty (as suggested in linear programming under uncertainty by Beale (1) and Dantzig (8)) or with a specified probability only (as suggested in chance-constrained programming by, for example, Charnes, Cooper and Symonds (5)).

While all these situations might be considered within the framework of this study, we assume that HQ allows for the probability distributions of stochastic elements by preferring decisions which maximize the expected value E(z) of the firm's horizon value and restrict the probability that any one of the periodic budgetary constraints will be violated by the realization of the decisions to specified risk levels.

Hillier (12) and Naslund (24) have discussed such decision rules for capital budgeting problems with regard to a decision maker's utility function. Their introduction into a mathematical programming problem has been formulated by Charnes, Cooper and Symonds (5) in an approach called chance-constrained programming in which constraints must be satisfied with a certain tolerance probability.

If $(1-\alpha_t)$, $t=1,..,T$, denote risk levels as specified by HQ, then HQ's decision problem can be formulated as the chance-constrained programming problem:

$$\text{maximize} \quad E(z_{H2}) = \sum_{k=1}^{K} E(\bar{a}_k)x_k + v_T - w_T$$

$$\text{subject to:} \quad \text{Prob}\ \left(-\sum_{k=1}^{K} \bar{a}_{1k}\bar{x}_k + v_1 - w_1 \le \bar{D}_1\right) \ge \alpha_1 \tag{2}$$

$$\text{Prob}\ \left(-\sum_{k=1}^{K} \bar{a}_{tk}\bar{x}_k - (1+\bar{r}_{t-1})v_{t-1} + v_t + (1+\bar{r}'_{t-1})w_{t-1} - w_t \le \bar{D}_t\right) \ge \alpha_t \quad (t=1,..,T)$$

$$0 \le \bar{x}_k \le 1 \quad (k=1,..,K)$$

$$v_t,\ w_t \ge 0 \quad (t=1,..,T)$$

$$\bar{a}_k,\ \bar{a}_{tk},\ \bar{D}_t \ \text{stochastic} \quad (t=1,..,T;k=1,..,K)$$

On the basis of this approach to its decision problem HQ may expect a better solution for the firm's capital budgeting problem which accords with its preferences, if it knew, in addition, the divisions' information about divisional opportunities. It is assumed that the divisions' information about the a_k and a_{tk} $(t=1,..,T;\ k=1,..,K)$ in the problem formulation (1), i.e., about their opportunities, is represented by vectors $\bar{\bar{a}}_k$ and $\bar{\bar{a}}_{tk}$ $(t=1,..,T;\ k=1,..,K)$. The elements of these vectors represent either the true values or random variables as the divisions' stochastic estimates of the horizon values of post horizon cash flows and of the periodic net cash flows that are associated with the realization of single projects.

In the following sections we discuss a situation where HQ may want to acquire this information by communicating with the divisions.

3. The Stochastic Communication Process

It is assumed that HQ may improve its information about divisional opportunities by iteratively exchanging information with the divisions in a process which, in principle, is organized as formulated in the decomposition principle of Dantzig and Wolfe (7). HQ "prices" the firm's capital assets whereas the divisions respond with proposals for divisional optimal use of these assets and the resulting output in terms of the firm's horizon value, i.e., its objective function value.

However, the proposed procedure differs from the decomposition principle in so far as

a) HQ's and the divisions' information about the firm's capital budgeting problem is stochastic and

b) the divisions' are supposed to inform HQ about the uncertainty in the proposals which they submit.

Denote by b_t^q (t=1,..,T) the periodic marginal horizon values ("prices") of the firm's capital assets as determined by HQ in iteration q of the communication process. Then each division k (k=1,..,K) might compute its proposal for the use of the firm's capital assets on the basis of the optimal solution $[x_k, u_{tk}(t=1,..,T)] = [x_k^q, u_{tk}^q(t=1,..,T)]$ of a programming problem that can be formulated mathematically as:

$$\text{maximize} \quad z_{D1} = E\left(\bar{\bar{a}}_k x_k + \sum_{t=1}^{T} b_t^q u_{tk}\right)$$

$$\text{subject to:} \quad -u_{tk} + \bar{\bar{a}}_{tk} x_k = 0 \qquad (t=1,..,T) \tag{3}$$

$$0 \le x_k \le 1$$

$$\left.\begin{array}{l} u_{tk} \text{ unrestricted in sign} \\ \bar{\bar{a}}_k,\ \bar{\bar{a}}_{tk} \text{ stochastic} \end{array}\right\} \qquad (t=1,..,T)$$

Define $\overset{+}{g}{}^q_k$ by the relationship $\overset{+}{g}{}^q_k = E(\bar{\bar{a}}_k)x^q_k$ and g^q_{tk} by $g^q_{tk} = u^q_{tk}$ $(t=1,\ldots,T)$. Then division k's proposal which it submits to HQ at iteration q of the communication process has the form $[\overset{+}{g}{}^q_k, g^q_{tk}(t=1,..,T)]$.

It is assumed that HQ adds the divisional proposals to its initial information about divisional opportunities and revises its programming problem according to rules that have been developed for the formulation of the master problem in the decomposition principle. After $q=Q$ information exchanges between HQ and the divisions, HQ's programming problem for the computation of revised "prices" for the firm's capital assets might be formulated in its probabilistic form as:

$$\text{maximize} \quad E(z_{H3}) = \sum_{k=1}^{K} E(\bar{a}_k)\bar{x}_k + \sum_{k=1}^{K}\sum_{q=2}^{Q} E(\overset{+}{g}{}^q_k)\,\lambda^q_k + v_T - w_T \tag{4a}$$

$$\text{subject to:} \quad \text{Prob}\left(-\sum_{k=1}^{K}\bar{a}_{1k}\bar{x}_k - \sum_{k=1}^{K}\sum_{q=2}^{Q} g^q_{1k}\,\lambda^q_k + v_1 - w_1 \le \bar{D}_1\right) \ge \alpha_1 \tag{4b}$$

$$\text{Prob}\left(-\sum_{k=1}^{K}\bar{a}_{tk}\bar{x}_k - \sum_{k=1}^{K}\sum_{q=2}^{Q} g^q_{tk}\,\lambda^q_k - \bar{r}_{t-1}v_{t-1} + v_t + \bar{r}'_{t-1}w_{t-1} - w_t \le \bar{D}_t\right) \ge \alpha_t \quad (t=2,..,T) \tag{4c}$$

$$I\bar{x}_k - \lambda^1_k \le 0 \quad (k=1,..,K) \tag{4d}$$

$$\lambda^1_k + \sum_{q=2}^{Q}\lambda^q_k = 1 \quad (k=1,..,K) \tag{4e}$$

$$\left.\begin{array}{l} \bar{x}_k,\ v_t,\ w_t,\ \lambda^1_k,\ \lambda^q_k \quad \ge 0 \\ \overset{+}{g}{}^q_k,\ g^q_{tk},\ \bar{a}_k,\ \bar{a}_{tk},\ \bar{D}_t \ \text{stochastic} \end{array}\right\} \quad (k=1,..,K;\ t=1,..,T;\ q=2,..,Q)$$

While the variables λ_k^q (q=2,..,Q;k=1,..,K) in (4) determine the realization of divisional proposals in the solution of HQ's programming problem, the variables λ_k^1 (k=1,..,K) can be interpreted as representing divisional proposals where none of the divisional decision variables are positive.

The formulation of the constraints (4d) is aimed at allowing HQ to realize the portions $\lambda_k^1 \bar{x}_k$ of its decision vectors $\bar{x}_k$ (k=1,..,K) which, as a supplement to the realization of divisional proposals, do not violate divisional constraints. It should be noted that this formulation does not allow HQ to realize its decision vectors x_k (k=1,..,K) to an extent where all divisional constraints are binding. Instead, it limits the realization of its decision vectors to portions defined for each k=1,..,K by the tightest periodic constraint in the divisional problems. This limitation of HQ's "flexibility" is implied in the communication rules of the decomposition principle and reflects HQ's limited information about divisional constraints.

By using an identity matrix I in the constraints (4d), it is assumed that HQ's decision vectors represent single projects or aggregates comprised of identical portions of different projects. In situations where these assumptions do not hold, the matrix should be adjusted accordingly.[3]

4. Realization of the Communication Process

For the realization of the iterative communication process it has to be converted into an equivalent deterministic form. Furthermore, the deterministic process has to be tractable and must allow (finite) convergence to an optimal or near optimal solution of HQ's planning problem. The conversion into a deterministic form requires

(a) the transformation of HQ's planning problem (4) and the divisions' planning problems (3) into deterministic equivalents and

(b) the specification of the (deterministic) information that has to be exchanged between HQ and divisions.

Both tasks are interrelated and can only be solved simultaneously.

In the following sections we will develop a deterministic form of the stochastic communication process which allows the application of convergence properties that are known for the decomposition principle (see, for example, (2), (6), (7), (18), (19), or (28)). The deterministic forms of HQ's and the divisions' planning problems can be regarded as a "master problem" and "subproblems" in a decomposition procedure aimed at solving a nonlinear optimization problem. However, it will be shown that this problem fulfills the requirements that have been formulated by Dantzig and Wolfe (7) to ensure convergence of the iterative solution process.

4.1 A Deterministic Equivalent for HQ's Programming Problem

HQ's programming problem (4) can be converted to a deterministic equivalent form by replacing the probability constraints by deterministic equivalents.

Let us consider the net cash flow s_t in a single period t of (4) which is defined by

$$s_t = - \sum_{k=1}^{K} \bar{a}_{tk}\bar{x}_k - \sum_{k=1}^{K} \sum_{q=2}^{Q} g_{tk}^{q}\lambda_k^{q} - \bar{r}_{t-1}v_{t-1} + v_t + \bar{r}'_{t-1}w_{t-1} - w_t - \bar{D}_t \tag{5}$$

Denote by s_{tx} a realization of s_t for any possible values of the decision variables. Since it is assumed that at least some of the elements of the vectors $\bar{a}_{tk}$ (k=1,..,K) and of the g_{tk}^{q} (k=1,..,K;q=2,..,Q), $\bar{r}_{t-1}$, $\bar{r}'_{t-1}$ and $\bar{D}_t$ in the budgetary constraint for period t are random variables, s_{tx} is a random variable as well. Denote the expected value and variance of s_{tx} by $E(s_{tx})$ and $V(s_{tx})$, respectively.

Furthermore, assume that the functional form of the probability distribution for s_{tx} is known, and that the fractiles of this distribution are completely determined by its mean and variance. Let F(h) denote the cumulative distribution function of the standardized variable $h=(s_{tx}-E(s_{tx}))/\sqrt{V(s_{tx})}$. Define $h_{\alpha t}$ by the relationship $F(h_{\alpha t}) = \alpha_t$. Then the deterministic equivalent form of the probability constraint in period t for specified values of the decision variables can be formulated as

$$E(s_{tx}) + h_{\alpha t} \sqrt{V(s_{tx})} \leq 0 \tag{6}$$

For the representation in HQ's programming problem (4), we use an approach which has been developed basically in linear programming with simple recourse (see, for example, Dantzig (9) or Ziemba (35))

and has been applied to capital budgeting problems with chance-constraints in (29). In this approach, the possible realizations of the random s_{tx} are considered explicitly in the problem formulation.

Define a"situation" c_t^i as a possible simultaneous realization $[\bar{a}_{tk}^i(k=1,..,K), g_{tk}^{qi}(k=1,..,K;q=2,..,Q), \bar{r}_{t-1}^i, \bar{r}'^i_{t-1}, \bar{D}_t^i]$ of the random elements of $\bar{a}_{tk}(k=1,..,K)$, $g_{tk}^q(k=1,..,K;q=2,..,Q)$, $\bar{r}_{t-1}$, $\bar{r}'_{t-1}$ and $\bar{D}_t$. Let C_t be a set of a finite number m of possible situations which are assumed to be known with their probabilities $p_t^i(i=1,..,m)$.

For any specification of the decision variables in (5) each situation c_t^i $(i=1,..,m)$ defines a possible realization s_{tx}^i $(i=1,..,m)$ of the random s_{tx}. As the values of the decision variables are not known explicitly, the values of the s_{tx}^i $(i=1,..,m)$ are not either. However, they can be determined simultaneously with the determination of an optimal decision vector by introducing variables s_t^i $(i=1,..,m)$ into HQ's programming problem such that $s_t^i = s_{tx}^i$ for any specification of the decision variables in the problem.

The deterministic equivalent form of the probability constraint in period t for an unspecified decision vector can then be formulated as

$$\sum_{i=1}^{m} p_t^i s_t^i + h_{\alpha t} \sqrt{\sum_{i=1}^{m} p_t^i (s_t^i - \sum_{j=1}^{m} p_t^j s_t^j)^2} \leq 0 \tag{7}$$

The incorporation of variables s_t^i $(i=1,..,m)$ into the periodic budgetary constraints of HQ's programming problem requires the explicit consideration of each possible situation in period t $(t=1,..,T)$. This might result in very large programming models even if the number m of possible situations in a single period is rather small. It has been suggested in (29) to reduce the size of the problem by selecting a random

sample of sequences of situations for consideration in the programming problem. However, in this case the $h_{\alpha t}$ in (7) should be adjusted accordingly (see (29)). By explicitly considering different situations, the probability constraints for, as an example, period t=1 can be formulated mathematically as:

$$- \sum_{k=1}^{K} \begin{bmatrix} \bar{a}^{1}_{1k}\bar{x}_k \\ \bar{a}^{2}_{1k}\bar{x}_k \\ \cdot \\ \cdot \\ \cdot \\ \bar{a}^{m}_{1k}\bar{x}_k \end{bmatrix} - \sum_{k=1}^{K} \sum_{q=2}^{Q} \begin{bmatrix} g^{q1}_{1k}\lambda^{q}_{k} \\ g^{q2}_{1k}\lambda^{q}_{k} \\ \cdot \\ \cdot \\ \cdot \\ g^{qm}_{1k}\lambda^{q}_{k} \end{bmatrix} + \begin{bmatrix} v_1 \\ v_1 \\ \cdot \\ \cdot \\ \cdot \\ v_1 \end{bmatrix} - \begin{bmatrix} w_1 \\ w_1 \\ \cdot \\ \cdot \\ \cdot \\ w_1 \end{bmatrix} + \begin{bmatrix} s^{1}_{1} & & & 0 \\ & s^{2}_{1} & & \\ & & \ddots & \\ 0 & & & s^{m}_{1} \end{bmatrix} = \begin{bmatrix} \bar{D}^{1}_{1} \\ \bar{D}^{2}_{1} \\ \cdot \\ \cdot \\ \cdot \\ \bar{D}^{m}_{1} \end{bmatrix} \qquad (8a)$$

$$\sum_{i=1}^{m} p^{i}_{1} s^{i}_{1} + h_{\alpha 1} \sqrt{\sum_{i=1}^{m} p^{i}_{1}\Big(s^{i}_{1} - \sum_{j=1}^{m} p^{j}_{1} s^{j}_{1}\Big)^{2}} \le 0 \qquad (8b)$$

$$s^{i}_{1} \ (i=1,\ldots,m) \text{ unrestricted in sign} \qquad (8c)$$

A similar transformation can be applied to all probability constraints of HQ's planning problem. It has been shown elsewhere (see Hillier (14) and Kataoka (17) that for continuous decision variables and $h_{\alpha t} \ge 0$ the deterministic equivalent forms of the probability constraints are convex. Furthermore, convex separable and linear approximations have been developed (see (14) or (29)). Consequently, the deterministic equivalent of HQ's problem is a convex programming problem which can be solved by means of convex programming algorithms or, when using a linear approximation, by any version of the simplex method.

4.2 The Computation of Divisional Proposals

For the computation of divisional proposals that can be used by HQ in its deterministic programming problem, the divisions' programming problems (3) have to be transformed accordingly. The transformation must allow the determination of the possible realizations of the random variables (in the divisions' proposals) that correspond to the various situations c_t^i (t=1,..,T; i=1,..,m).

Denote by b_t^{qi} (t=1,..,T; i=1,..,m) the marginal horizon values of the firm's capital assets in situation c_t^i (t=1,..,T; i=1,..,m) which have been determined by HQ in iteration q of the communication process as the optimal values of the dual variables that correspond to the linear budgetary constraints in its deterministic programming problem (numbered (8a) in the example outlined for period t=1).

Furthermore, denote by $\bar{\bar{a}}_{tk}^{i}$ (t=1,..,T; i=1,..,m) the realizations of the random $\bar{\bar{a}}_{tk}$ that correspond to the different situations c_t^i (t=1,..,T; i=1,..,m). Then division k might compute its proposal for the use of the firm's capital assets by solving a linear programming problem that can be formulated mathematically as:

$$\text{maximize} \quad z_{D2} = E(\bar{\bar{a}}_k)x_k + \sum_{t=1}^{T}\sum_{i=1}^{m} p_t^i(b_t^{qi}u_{tk}^i)$$

$$\text{subject to:} \quad \begin{bmatrix} \bar{\bar{a}}_{tk}^{1}x_k \\ \bar{\bar{a}}_{tk}^{2}x_k \\ \cdot \\ \cdot \\ \cdot \\ \bar{\bar{a}}_{tk}^{m}x_k \end{bmatrix} - \begin{bmatrix} u_{tk}^1 & & & 0 \\ & u_{tk}^2 & & \\ & & \ddots & \\ 0 & & & u_{tk}^m \end{bmatrix} = 0 \quad (t=1,..,T) \qquad (9)$$

$$0 \le x_k \le 1$$

$$u_{tk}^i \text{ unrestricted in sign } (t=1,..,T;\ i=1,..,m)$$

4.3 Organization of the Iterative Information Exchange

The application of rules using the decomposition principle would result in a communication scheme where during any iteration q

a) HQ computes the marginal horizon values b_t^{qi} (t=1,..,T; i=1,..,m) of the firm's net cash flows that correspond to the distinguished situations c_t^i (i=1,..,m) in each period t (t=1,..,T) using its deterministic problem whereas

b) a division k (k=1,..,K) responds with a proposal [$\overset{+q}{g}_k, g_{tk}^{qi}$(t=1,..,T; i=1,..,m)], i.e. a proposal that specifies the division's use of capital assets in each of the periodic c_t^i (i=1,..,m).

However, the realization of this scheme implies that HQ and the divisions are able to associate the information which they receive with the situations c_t^i (t=1,..,T; i=1,..,m).

While this should not be a problem in mathematical decomposition it might prove to be difficult in a communication process between HQ and divisions. In such cases, depending on the specific situation, a variety of alternative possibilities for aiding HQ and the divisions in associating communicated information with different situations could be formulated.

As an example, HQ might inform the divisions about the b_t^{qi} (t=1,..,T; i=1,..,m) by reporting the compounded information [$g_{tk}^{(q-1)i}, b_t^{qi}$] with i=1,..,m and t=1,..,T to each division k (k=1,..,K). The $g_{tk}^{(q-1)i}$ (t=1,..,T; i=1,..,m) are assumed to represent parts of a divisional proposal that has been submitted to HQ in an earlier iteration of the communication process and has been recorded by division k.

On the other hand, the divisions might link their new proposals g_{tk}^{qi} to the b_t^{qi} by reporting to HQ the compounded information $[g_{tk}^{qi}, b_t^{qi}]$ with $i=1,..,m$ and $t=1,..,T$.

However, the amount of information that has to be exchanged in this scheme between HQ and divisions at each iteration of the communication process can be reduced considerably. HQ and divisions might inform each other about the probability distribution of the random variables in their reports and the relation to the distinguished situations by communicating variances V and covariances COV. In this case, HQ is supposed to report to each division k the variances V (b_t^q), $t=1,..,T$, and covariances COV $(g_{tk}^{(q-1)}, b_t^q)$, $t=1,..,T$, while the division responds with variances V (g_{tk}^q), $t=1,..,T$, and covariances COV (g_{tk}^q, b_t^q), $t=1,..,T$. Then HQ and divisions could generate the individual realizations of the random g_{tk}^q and b_t^q and their linkage to the different situations from the variances and covariances by using techniques that have been developed by, for example, Moonan (23) or Scheuer and Stoller (26).[4]

The communication scheme could then be formulated in the following way:

1. HQ computes b_t^{qi} $(t=1,..,T;\ i=1,..,m)$ by solving its deterministic programming problem.
2. HQ computes and reports to each division k $(k=1,..,K)$ variances V (b_t^q) and covariances COV $(g_{tk}^{(q-1)}, b_t^q)$, $t=1,..,T$.
3. The divisions generate b_t^{qi} $(t=1,..,T;\ i=1,..,m)$ and determine new proposals by solving the programming problems (9).
4. Each division k $(k=1,..,K)$ computes and reports to HQ variances $V(g_{tk}^q)$ and covariances $COV(g_{tk}^q, b_t^q)$, $t=1,..,T$.

5. HQ generates the g_{tk}^{qi} ($t=1,..,T$; $i=1,..,m$) for each division k ($k=1,..,K$), incorporates them into its programming problem and continues with iteration $q=q+1$.

The communication process might be terminated by HQ if it arrives at a solution it considers satisfactory close to the optimal solution of its planning problem or which it does not expect to get improved substantially in further iterations of the process.[5]

It should be noted that in this scheme the information HQ is supposed to report to the K different divisions is no longer identical as it is in the decomposition principle. While the variances still are, the covariances submitted by HQ are different for different divisions.

4.4 Properties of the Communication Process

In the context of the decomposition principle of Dantzig and Wolfe (7), the deterministic programming problems that are solved by HQ and the divisions during each iteration of the communication process can be regarded as master problem and subproblems derived from an optimization problem that could be formulated mathematically in its probabilistic form as:

$$\text{maximize} \quad E(z_O) = \sum_{k=1}^{K} E(\bar{a}_k)\bar{x}_k + E(\bar{\bar{a}}_k)x_k - v_T - w_T \tag{10a}$$

$$\text{subject to:} \quad \text{Prob}\left(-\sum_{k=1}^{K} \bar{a}_{1k}\bar{x}_k - \sum_{k=1}^{K} u_{1k} + v_1 - w_1 \le \bar{D}_1\right) \ge \alpha_1 \tag{10b}$$

$$\text{Prob}\left(-\sum_{k=1}^{K} \bar{a}_{tk}\bar{x}_k - \sum_{k=1}^{K} u_{tk} - (1+\bar{r}_{t-1})v_{t-1} + v_t + (1+\bar{r}'_{t-1})w_{t-1} - w_t \le \bar{D}_t\right) \ge \alpha_t \quad (t=2,..,T) \tag{10c}$$

$$I\bar{x}_k - Ix_k \le 0 \qquad (k=1,..,K) \tag{10d}$$

$$u_{tk} - \bar{\bar{a}}_{tk}x_k = 0 \qquad (k=1,..,K; t=1,..,T) \tag{10e}$$

$$0 \le x_k \le 1 \qquad (k=1,..,K) \tag{10f}$$

$$\left.\begin{array}{l} u_{tk} \text{ unrestricted in sign} \\ \bar{a}_k,\ \bar{\bar{a}}_k,\ \bar{a}_{tk},\ \bar{\bar{a}}_{tk},\ \bar{D}_t \text{ stochastic} \\ \bar{x}_k,\ v_t,\ w_t \ge 0 \end{array}\right\} \begin{array}{c} (k=1,..,K; \\ t=1,..,T) \end{array} \tag{10g}$$

It is easy to see that the deterministic formulation of (10) has a block-angular structure[6] in its matrix where the deterministic forms of the constraints (10b) - (10d) represent the "common" constraints whereas the diagonal submatrices refer to the constraints (10e) and (10f).

Furthermore, the parts of the common constraints and the objective function that correspond to the divisional constraints (10e) and (10f), i.e., the parts that include the model variables u_{tk} and x_k $(t=1,..,T; k=1,..,K)$ in the deterministic equivalent of (10), are linear.

However, the linearity in constraints and objective function and the identification of a block-angular structure in the matrix qualify an optimization problem for a successful application of the decomposition principle. Therefore, the convergence properties that have been specified for the decomposition principle do hold for the outlined communication process as well.

5. Consideration of Cheating Divisions

The consideration of divisions who are submitting incorrect information about their opportunities while communicating with HQ has been of major concern with regard to the realization of communication processes. In situations where HQ is suspecting divisional cheating, it might want to determine "forecasts" for the proposals it expects from the divisions. By comparing the forecasts with the proposals reported by the divisions, HQ might get a basis for a revision of the divisions' proposals if its confidence in the correctness of the divisions' response is limited.

Now, let us assume that HQ is able to quantify stochastic information about the divisions' opportunities, i.e. quantify elements of the vectors $\bar{a}_k$ and $\bar{a}_{tk}$ $(t=1,..,T;\ k=1,..,K)$ in its programming problem (4). This would allow the formulation of approximations for the divisions' programming problems (3) and, subsequently, the determination of stochastic forecasts for the divisional proposals HQ expects in response to the prices it suggested for the use of the firm's capital assets. Furthermore, let us assume that HQ's confidence in the correctness of the divisions' proposals is limited and that it wants to revise the proposals before incorporating them into its programming problem.

As an example, such a revision might be based on the Theorem of Bayes if HQ is able to express its confidence in the divisions' proposals in terms of conditional probabilities.

Denote by $[f_k^q, f_{tk}^q (t=1,..,T)]$ HQ's forecast for the proposal $[g_k^q, g_{tk}^q (t=1,..,T)]$ that has been reported by division k in iteration q of the communication process. Furthermore, denote the possible realizations of the random variables f_k^q and f_{tk}^q $(t=1,..,T)$ in HQ's forecast that correspond to the distinguished situations in HQ's deterministic programming problem by f_k^{qi} $(i=1,..,m)$ and, for any $t=1,..,T$, by f_{tk}^{qi} $(i=1,..,m)$. Denote the probability of any of these possible realizations by $P(f_k^{qi})$ or $P(f_{tk}^{qi})$, respectively.

Suppose now that HQ's confidence in the correctness of the divisions' proposals is expressed by conditional probabilities $P(g_k^q/f_k^{qi})$, $i=1,..,m$, and, for any of the random periodic net cash flows, by $P(g_{tk}^q/f_{tk}^{qi})$, $i=1,..,m$.

Then HQ might revise the probability distributions of the random variables in its forecast by applying the Theorem of Bayes. As an example, HQ might compute the revised probability distribution for any of the random f_{tk}^q which it wants to incorporate into its programming problem as:

$$P(f_{tk}^{qi}/g_{tk}^q) = \frac{P(f_{tk}^{qi}) \cdot P(g_{tk}^q/f_{tk}^{qi})}{\sum_{j=1}^{m} P(f_{tk}^{qj}) \cdot P(g_{tk}^q/f_{tk}^{qi})} \qquad (i=1,..,m) \qquad (12)$$

The random forecast $[f_k^q, f_{tk}^q (t=1,..,T)]$ with the revised probability distributions for the different random variables is then incorporated by HQ in its programming problem as division k's "true" response to HQ's computed prices for the use of the firm's capital assets in iteration q of the communication process. However, in situations where HQ allows for cheating divisions in this way, the convergence properties of the decomposition principle might no longer apply to the iterative communication process.

6. Conclusion

In this paper a procedure has been presented which might be used by HQ in a multidivisional firm to arrive at a solution for a stochastic capital budgeting problem in situations where its information about the divisions' opportunities is limited but where it may improve its initial information by iteratively communicating with the divisions.

It has been assumed that HQ is willing to accept a calculated risk that periodic budgetary constraints might be violated. This attitude has been introduced in its programming problem by means of chance-constraints. The organization of the iterative information exchange between HQ and divisions is organized in the procedure according to rules that could be interpreted as a variation of the decomposition principle.

It can easily be shown that the procedure can be applied in situations where HQ and the divisions have to consider, in addition, stochastic or deterministic financial and physical constraints on the realization of their decision variables.

Footnotes

1. These decision-making systems are sometimes referred to as coordinable. See, for example Mesarovic et. al. (22) or Jennergren (15).

2. For the formulation of capital budgeting problems as optimization problems see Weingartner (34). The maximization of a firm's horizon value in the objective function has primarily been suggested by Charnes, Cooper and Miller (4).

3. See, for example, Freeland and Schiefer (10).

4. These techniques are based on the assumption of normality.

5. For the determination of a maximum possible improvement see Lasdon (18).

6. See Lasdon (18).

7. See, for example, Raiffa (25).

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Abstract

In this paper we formulate an iterative solution procedure for stochastic capital budgeting problems in multidivisional firms where headquarters must arrive at an optimal divisional allocation of capital assets. It is assumed that headquarters allows for its responsibility by accepting a calculated risk that periodic budgetary constraints might be violated. This attitude towards risk is modelled via the use of chance constraints. The organization of the information exchange between headquarters and divisions is basically derived from the decomposition principle but possible variations are discussed. The proposed procedure allows the consideration of divisions who are cheating while submitting divisional information to headquarters.